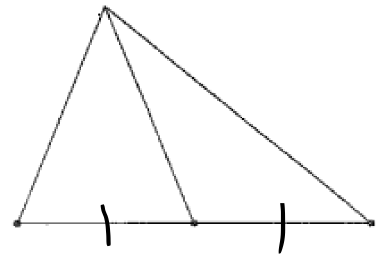
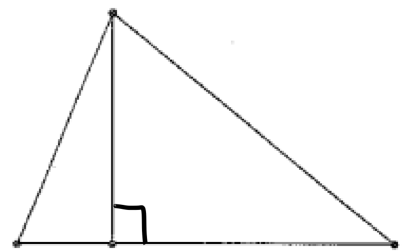
Medians and Altitudes

Vocabulary!!

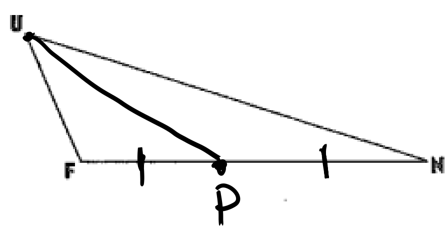
- **Median** – a segment whose endpoints are the vertex of the triangle and the midpoint of the opposite side.



- **Altitude** – the segment that passes through the vertex of a triangle and is perpendicular to the opposite side or to the line that contains the opposite side.



Ex 1 **Given:** $\overline{UP}$ is a median



Sketch the given in the picture

What does the given tell you?

$\overline{FP} \cong \overline{PN}$

Label this in your picture.

Solve for missing variable and find FN. FN = 70

Given: FP = x + 15 and PN = 4x – 45

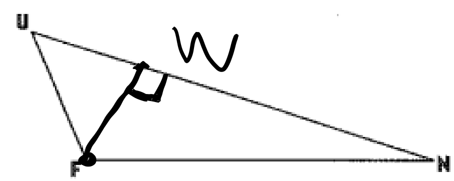
$x + 15 = 4x - 45$

$60 = 3x \quad x = 20$

$20 + 15 = 35$

$4(20) - 45 = 35$

Ex 2 **Given:** $\overline{FW}$ is an altitude



Sketch the given in the picture

What does the given tell you?

$\overline{FW} \perp \overline{UN}$

Label this in your picture.

Solve for missing variable.

Given: $m\angle UWF = (15x - 30)^\circ$

$15x - 30 = 90$

$15x = 120$

$x = 8$

True or False??? Why??

- If a segment is both a median and an altitude, then it must be also be a perpendicular bisector.

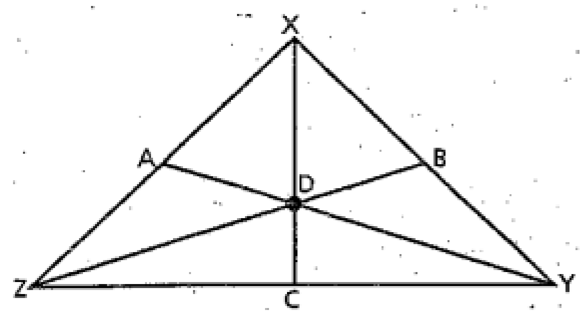
True - altitude is ⊥ and median goes through mdpt

- If a segment is a perpendicular bisector, then it must be both a median and an altitude.

False - perpendic ⊥ not always thru vertex
- altitude not always thru midpoint

Medians of a Triangle

- The centroid is the point at which all three **medians** are concurrent (where they intersect).
- center of gravity in the triangle



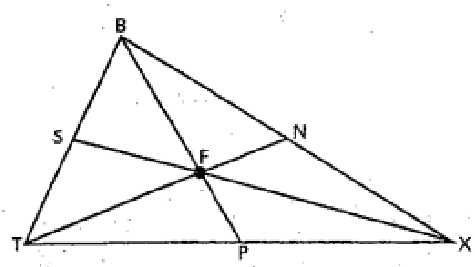
<u>Concurrency of Medians of a Triangle Theorem</u>

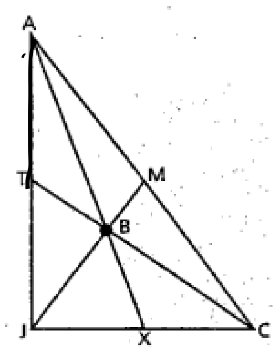
The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

The medians of ΔXYZ meet at point D and

XD = $\frac{2}{3}XC$ *YD* = $\frac{2}{3}YA$ *ZD* = $\frac{2}{3}ZB$

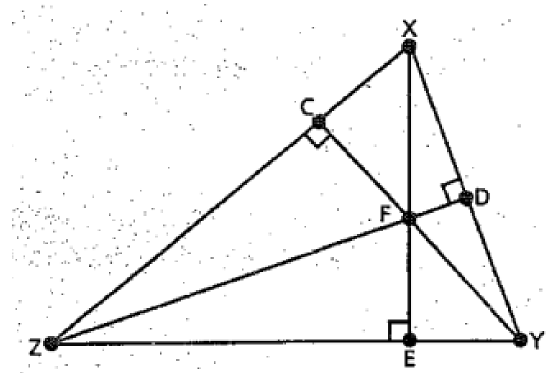
Use the diagrams to answer the questions.





1. Name the medians. $\overline{BP}$ $\overline{TN}$ $\overline{XS}$
2. What is the centroid of this triangle? F
3. Is $\overline{BX}$ a median or a side of this triangle? side
4. Is $\overline{PT} \cong \overline{BS}$? no
5. Is $\overline{PT} \cong \overline{PX}$? yes
6. Name the medians. $\overline{AX}$ $\overline{TC}$ $\overline{MJ}$
7. What is the center of gravity? B
8. Is $\overline{TC}$ a median or a side of this triangle? median
9. Is $\overline{AT} \cong \overline{TJ}$? yes
10. Is $\overline{MC} \cong \overline{MA}$? yes

Altitudes of a Triangle



- The orthocenter is the point of concurrency of the lines containing the three **altitudes** of the triangle.

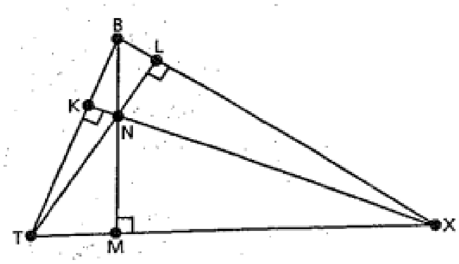
Concurrency of Altitudes of a Triangle Theorem

The lines containing the altitudes of a triangle are concurrent.

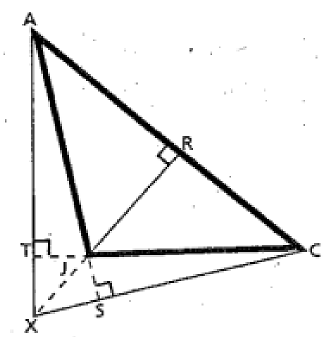
If $\overline{XE}, \overline{YC}$ and $\overline{ZD}$ are altitudes of ΔXYZ, then the lines containing these segments intersect at some point F.

Acute Δ: **inside** Right Δ: **on the Δ** Obtuse Δ: **outside**

Use the diagrams to answer the questions.



1. Name the altitudes. $\overline{MB}$ $\overline{XK}$ $\overline{TL}$
2. What is the orthocenter of the triangle? N
3. What is perpendicular to $\overline{BL}$? $\overline{NL}$
4. What is perpendicular to $\overline{NM}$? $\overline{MX}$



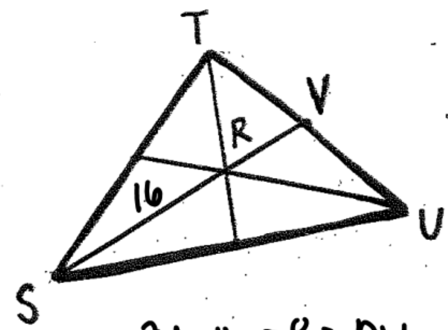
When a triangle is obtuse, two of the altitudes are outside of the triangle. Therefore the orthocenter is outside of the triangle.

5. Name the altitudes. $\overline{JR}$ $\overline{TA}$ $\overline{SC}$
6. What is the orthocenter of the triangle? X
7. What is perpendicular to $\overline{SC}$? AS
8. What is perpendicular to $\overline{RC}$? $\overline{JR}$

1) R is the centroid of $\triangle STU$

SR = 16. Find SV = 24

RV = 8



$SR = \frac{2}{3}SV$

$16 = \frac{2}{3}SV$

$\frac{3}{2}16 = SV \rightarrow$

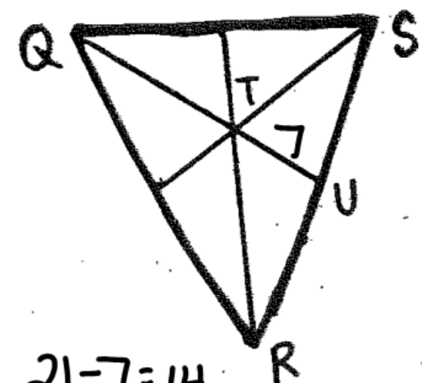
$\frac{48}{2} = SV \rightarrow SV = 24$

$24 - 16 = 8 = RV$

2) T is the centroid of $\triangle QRS$

TU = 7. Find QU = 21

QT = 14



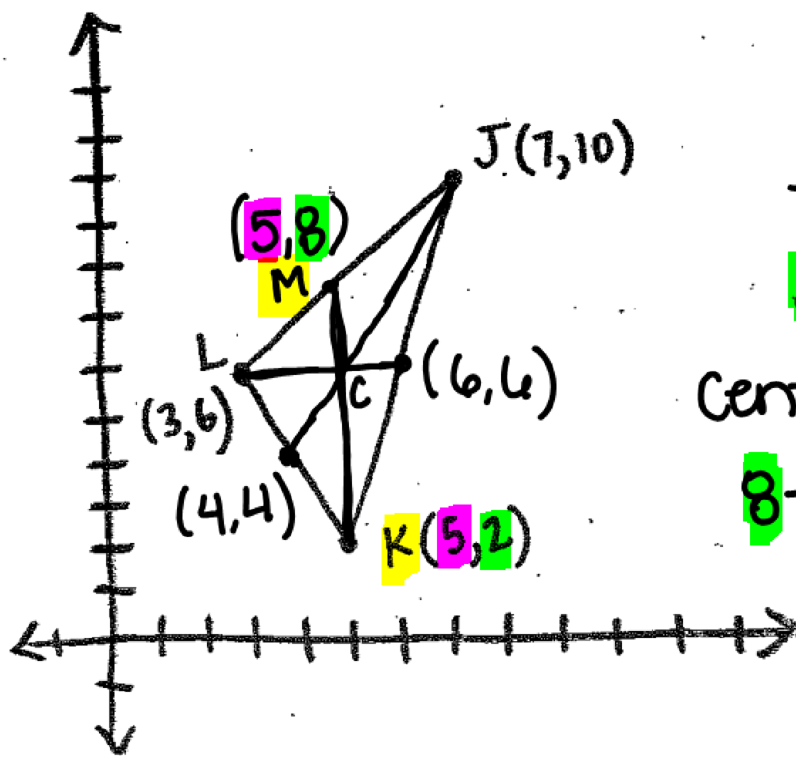
$TU = \frac{1}{3}QU$

$7 = \frac{1}{3}QU$

$21 = QU$

$21 - 7 = 14$

Find the coordinates of the centroid.



$\frac{2}{3}(6) = 4$

$2 + 4 = 6$

Centroid: (5, 6)

$8 - 2 = 6 = KM$